Fermi-Dirac Statistics

By:

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- Two key scientists behind the development of Fermi-Dirac statistics are Enrico Fermi and P.A.M Dirac.

Enrico Fermi

P.A.M Dirac

Fermi-Dirac Statistics

- It determines the statistical distribution of Fermions.
- Fermions are particles with half integral spin angular momentum and they obey Pauli's Exclusion Principle i.e no two particles can occupy same state at the same time.
- Examples of Fermions are: Electrons, protons, neutrons, neutrinos etc.

Fermi-Dirac Distribution Law

- The number of ways of distributing n_i particles among the g_i sublevels of an energy level is given by:

$$w(n_i, g_i) = \frac{g_i!}{n_i!(g_i - n_i)!} \ .$$

- The number of ways that a set of occupation numbers n_i can be realized is the product of the ways that each individual energy level can be populated:

$$W = \prod_i w(n_i, g_i) = \prod_i \frac{g_i!}{n_i!(g_i - n_i)!}.$$

- we wish to find the set of n_i for which *W(Thermodynamic Probability)* is maximized, subject to the constraint that there be a fixed number of particles, and a fixed energy.

$$f(n_i) = \ln(W) + \alpha\left(N - \sum n_i\right) + \beta\left(E - \sum n_i \epsilon_i\right).$$

- Using Stirling's approximation for the factorials and taking the derivative with respect to n_i, and setting the result to zero and solving for n_i yields the Fermi-Dirac population numbers:

$$n_i = \frac{g_i}{e^{\alpha+\beta\epsilon_i} + 1}.$$

Substituting $\beta=1/kT$

where k=Boltzmann's constant

$$n_i = \frac{g_i}{e^{\alpha} e^{\varepsilon_i/kT} + 1}$$

This is **Fermi-Dirac Distribution Law.**

The value of α can be calculated as per the conditions of a particular system.

Fermi-Energy :is the energy value upto which all energy states are filled at 0K and above which all the energy states are empty.This is given by:

$$E_F = \frac{h^2}{2m}(3n/8\pi V)^{2/3}$$

Where n=no.of conduction electrons

V=volume of the conductor

Fermi-Dirac distribution and the Fermi-level

The Fermi Energy function $f(E)$ specifies how many of the existing states at the energy E will be filled with electrons. The function $f(E)$ specifies, under equilibrium conditions, the probability that an available state at an energy E will be occupied by an electron. It is a probability distribution function.

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$

(2.7)

E_F = Fermi energy or Fermi level

k = Boltzmann constant = 1.38×10^{-23} J/K = 8.6×10^{-5} eV/K

T = absolute temperature in K

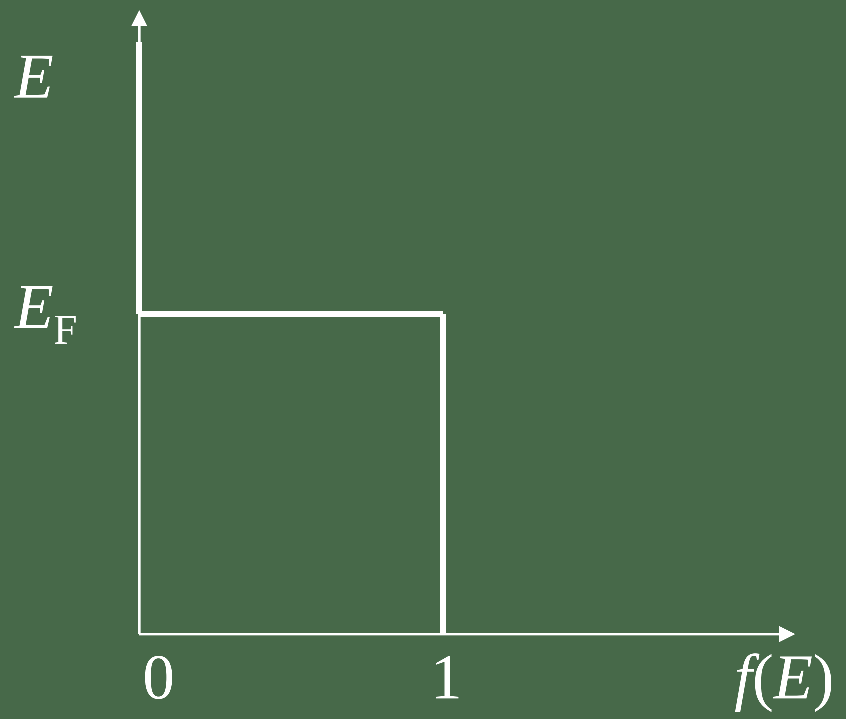
Fermi-Dirac distribution: **Consider $T \to 0$ K**

For $E > E_F$:

$$f(E > E_F) = \frac{1}{1 + \exp(+\infty)} = 0$$

For $E < E_F$:

$$f(E < E_F) = \frac{1}{1 + \exp(-\infty)} = 1$$



Fermi-Dirac distribution: **Consider $T > 0$ K**

If $E = E_F$ then $f(E_F) = ½$

If $E \geq E_F + 3kT$ then $\exp\left(\frac{E - E_F}{kT}\right) \gg 1$

Thus the following approximation is valid: $f(E) = \exp\left(\frac{-(E - E_F)}{kT}\right)$

i.e., most states at energies $3kT$ above E_F are empty.

If $E \leq E_F - 3kT$ then $\exp\left(\frac{E - E_F}{kT}\right) \ll 1$

Thus the following approximation is valid: $f(E) = 1 - \exp\left(\frac{E - E_F}{kT}\right)$

So, $1 - f(E)$ = Probability that a state is empty, decays to zero.

So, most states will be filled.

kT (at 300 K) = 0.025eV, E_g(Si) = 1.1eV, so $3kT$ is very small in comparison.

Temperature dependence of Fermi-Dirac distribution

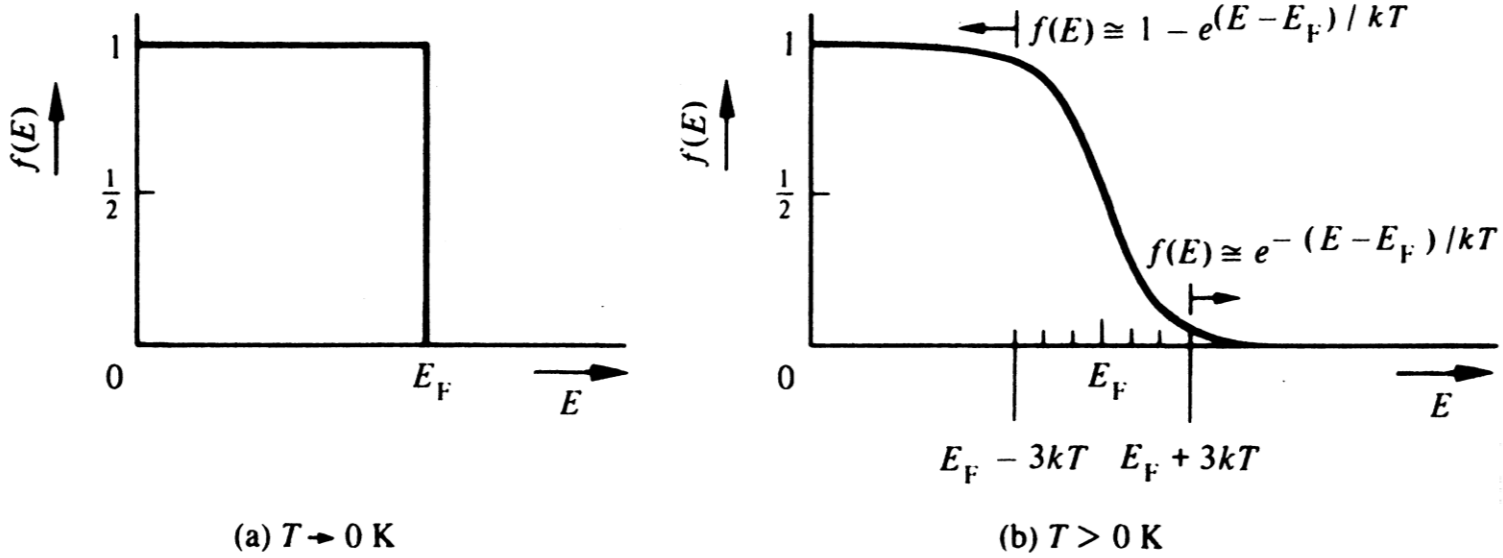
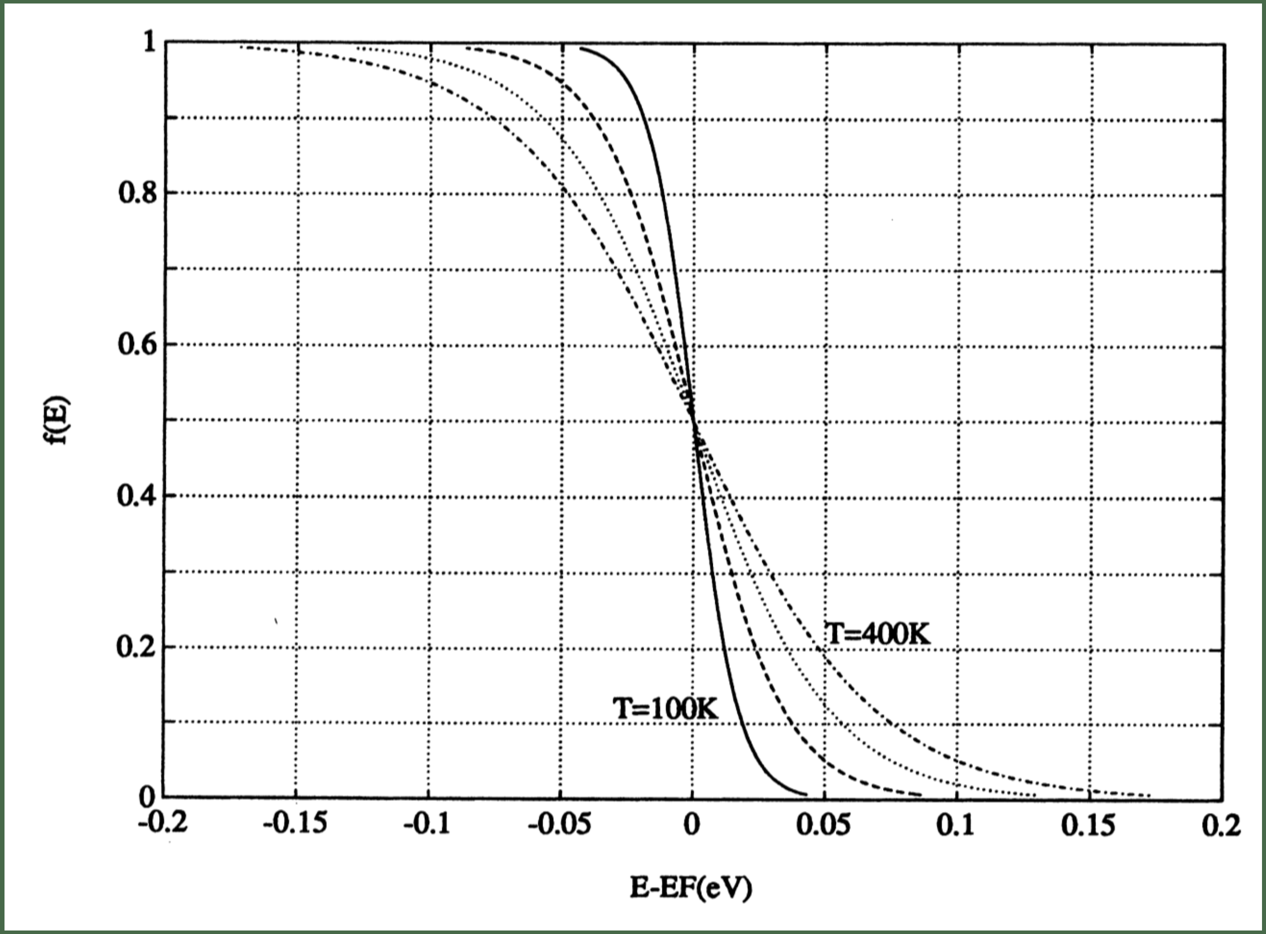


Figure 2.15

Variation of Fermi energy function with Temperature



Applications of Fermi-Dirac Statistics

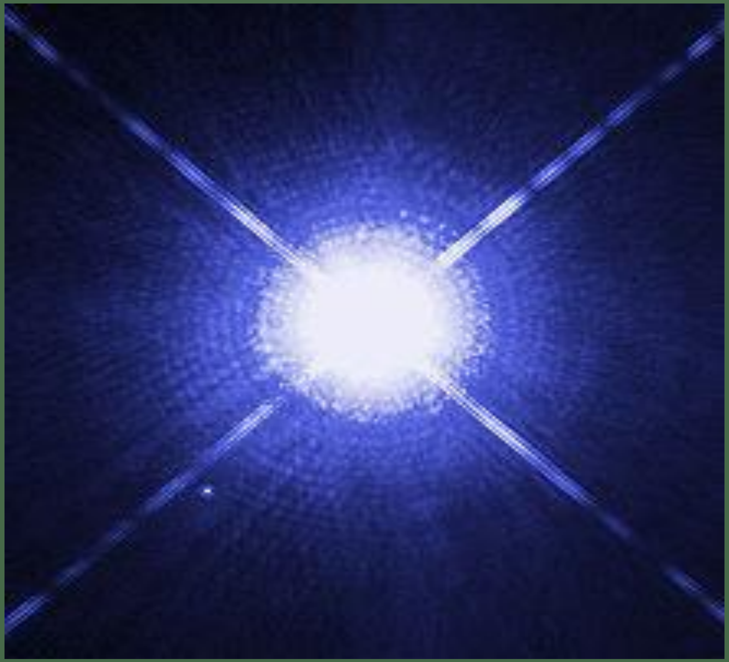
- The most important application of the F-D distribution law is in predicting the behaviour of free electrons inside conductors.
- The collection of these free electrons form a sort of gas known as **Fermi Gas**.
- Fermi-Dirac distribution law of electron energies is given by:

$$n(u)du = \frac{8\sqrt{2}\pi V m^{3/2}}{h^3} \frac{u^{1/2}du}{e^{\alpha+u/kT}+1}$$

As the temperature of the system is decreased,the energy of the system also decreases.The electrons tend to occupy lower energy states as the system is cooled.

Stability of White Dwarfs

- This is another important application of
  F-D statistics.
- White dwarf stars are stars of very small sizes(About size of earth),having masses 0.2-1.4 times the mass of sun,having high density and high surface temperatures(~10,000K to 30,000K).Due to such high temperature they appear white.



- White Dwarfs contain free electrons, protons, neutrons and other nuclei. These free protons or neutrons constitute Fermi gas.
- Since the pressure exerted by a gas of fermions is proportional to the Fermi energy, the pressure exerted by electrons inside a white dwarf is much higher than due to protons, neutrons and nuclei.
- This outward pressure due to free electrons acts against and balances the inward acting force of gravity and is largely responsible foe stability of white dwarfs.

Thank You